Research Note

Channel Model for Underwater Acoustic Propagation in the Indian Ocean Region (IOR)

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Background

Underwater acoustics is the study of the propagation of sound in water and the interaction of the acoustic (sound) waves with the water, its contents and its boundaries [1]. A lot of work have been done on underwater channel development from ray models to wave models. Existing models and theories can be applied to specific regions depending upon the parametres unique to the region. The objective of this research note is to review the existing channel models based on wave theory and direct the study to the applications and future scope in the Indian Ocean Region (IOR).

Sound Speed: Sound speed was studied first in the field of underwater acoustics as sound speed in water is same as refractive index in optics [2]. In 1687, Isaac Newton provided the first mathematical expression for sound in his book “Mathematical Principles of Natural Philosophy” [3], wherein he predicted that speed of sound as follows [4]:

$$c = \sqrt{\frac{P}{D}}$$

c = Speed of Sound
P = Pressure Acting on the Sound
D = Density of the medium

The formula was further enhanced by Laplace who suggested inclusion of bulk modulus instead of pressure, thus creating Newton-Laplace equation [5].

$$c = \sqrt{\frac{K}{D}}$$

c = Speed of Sound
K = Elastic Bulk Modulus
D = Density of the medium

Underwater acoustics entails the development and employment of acoustical methods to image underwater features, to communicate information via the oceanic waveguide, or to measure oceanic properties. Acoustic waves are the major source for carrying the underwater communication because electromagnetic waves are very easily attenuated and we cannot use the optical communication either due to the severe absorption of the optical waves in sea water [6]. Acoustic propagation in the underwater channel influenced by three factors: signal attenuation [7], multipath propagation [8], and low speed of sound propagation [9]. Although APMs can be classified according to the theoretical approach employed, the cross-connections that exist among the various approaches complicate a strict classification, or taxonomic, scheme [10].

Parameters of sound propagation are [10]:
SSP(Sound Speed Profile): Sound speed in the oceans depends on temperature, salinity, and pressure and has large seasonal and spatial variations [11].

Bathymetry: Bathymetry is the measurement of the depth of water in oceans, rivers, or lakes [12,13].

Absorption: When the sound propagates through the water, part of the energy is absorbed by the water and converted to heat [14].

Sea State: It is general condition of the state of the sea with respect to the wind and waves [15].

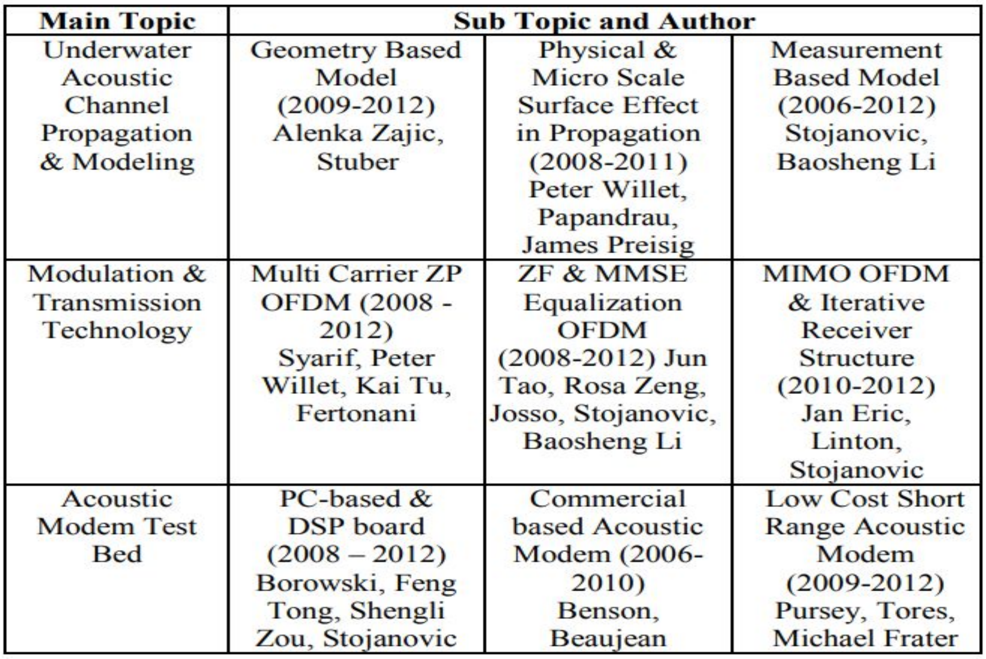
Bottom Composition: The outer rocky layer of the Earth includes about a dozen large sections called tectonic plates that are arranged like a spherical jigsaw puzzle floating on top of the Earth's hot flowing mantle. Many ocean floor features are a result of the interactions that occur at the edges of these plates [16].

Literature Review

The earliest attempts at modelling sound propagation in the sea were motivated by practical problems in predicting sonar performance in support of anti-submarine warfare (ASW) operations during World War II [17]. These early models used ray-tracing techniques derived from the wave equation to map those rays defining the major propagation paths supported by the prevailing marine environment [18]. These paths could then be used to predict the corresponding sonar detection zones. This approach was a forerunner to the family of techniques now referred to as ray theoretical solutions. An alternative approach, referred to as wave-theoretical solutions, was first reported by Pekeris (1948), who used the normal-mode solution of the wave equation to explain the propagation of explosively generated sound in shallow water [10]. As modelling technology matured over the intervening decades, the attendant sophistication has complicated the simple categorization of ray versus wave models. The terminology is still useful in distinguishing those models based principally on ray-tracing techniques from those using some form of numerical integration of the wave equation [10]. Occasionally, a mixture of these two approaches is used to capitalize on the strengths and merits of each and to minimize weaknesses. Such combined techniques are referred to as hybrid approaches. Related developments in propagation modelling have been reviewed by Harrison (1989), McCammon (1991), Buckingham (1992), Porter (1993), and Dozier and Cavanagh (1993) [9]. Finite-element methods have also been used in underwater acoustics to treat problems requiring high accuracy (see Kalinowski (1979) for a good introduction to applications in underwater acoustics [10]).

The progress after 2000 can be referred from [19] and are briefly shown here:-

TABLE 1. UNDERWATER COMMUNICATION RESEARCH PROGRESS



Main Topic	Sub Topic and Author		
Underwater Acoustic Channel Propagation & Modeling	Geometry Based Model (2009-2012) Alenka Zajic, Stuber	Physical & Micro Scale Surface Effect in Propagation (2008-2011) Peter Willet, Papandrau, James Preisig	Measurement Based Model (2006-2012) Stojanovic, Baosheng Li
Modulation & Transmission Technology	Multi Carrier ZP OFDM (2008 - 2012) Syarif, Peter Willet, Kai Tu, Fertonani	ZF & MMSE Equalization OFDM (2008-2012) Jun Tao, Rosa Zeng, Josso, Stojanovic, Baosheng Li	MIMO OFDM & Iterative Receiver Structure (2010-2012) Jan Eric, Linton, Stojanovic
Acoustic Modem Test Bed	PC-based & DSP board (2008 – 2012) Borowski, Feng Tong, Shengli Zou, Stojanovic	Commercial based Acoustic Modem (2006-2010) Benson, Beaujean	Low Cost Short Range Acoustic Modem (2009-2012) Pursey, Tores, Michael Frater

Wave Equation

The objective is to derive the 1D simple linear wave equation for sound propagation in fluids. In reality, the acoustic equation is nonlinear and shows up only at high amplitudes therefore more complicated but in most real life applications the linear approximation model is good and applicable [20].

An acoustic wave is a type of mechanical wave where pressure variation propagates through a material. A local pressure change causes immediate fluid to compress which in turn causes additional pressure change and the wave propagates through disturbance caused in the fluid as a series of expansions and compressions. That is, Generation-> Transducer (piston for example) creates a particle displacement (which in turn has an associated pressure and density change). This change affects the immediately adjacent region, etc., so that the disturbance (wave) propagates [21].

A **uniform plane wave** has common phase and amplitude perpendicular to the direction of motion [22]. A **particle** is defined in terms of quantum mechanics. It may consist of many molecules (large enough to be considered as continuous medium), but its dimensions are small compared to the distances for significant changes in the acoustic parameters (for example, small enough to consider acoustic variables constant throughout particle volume) [22].

Parameters for wave propagation

Acoustic pressure: The difference between the pressure caused by a sound wave and the ambient pressure of the media the sound wave is passing through [23].

Particle displacement: The particles immediately to the right in front of the piston move with the piston as it oscillates back and forth. Elsewhere in the pipe, the particles oscillate back and forth, right and left, though they are not all moving in the same direction at the same time [24].

Particle velocity: Particle velocity has been a major consideration and it has been shown quite clearly that there is a threshold value of particle velocity below which no degradation occurs [25]. The value of this particle velocity for the aluminium oxide [26] was about 10 m/s and was influenced only slightly by particle size, target material [27], and particle impact angle [9] above about 15 degrees [22].

Particle acceleration: Acceleration is defined as the rate of change (or time derivative) of velocity [28].

Density changes: The greater the elasticity and the lower the density, the faster sound travels in a medium. The mathematical relationship is speed = (elasticity/density).

Velocity potential: A velocity potential is a scalar potential used in potential flow theory [29].

The linear wave equation derived from Euler's equation [Appendix A] is:

$$\frac{\partial^2 p}{\partial t^2} = c^2 \nabla^2 p$$

Where p is *acoustic pressure*, c is *speed of sound* and $\square^2$ is the *Laplacian operator* [30].

Further, Helmholtz equation can be derived from the wave equation y assuming harmonic solution to it [Appendix B]:

$$\nabla^2 \varphi + k^2 \varphi = 0$$

where $\square^2$ is the Laplacian operator[31], φ is the *time-independent potential function* and k is the *wave number*.

Solutions to the Helmholtz equation are summarized here:-

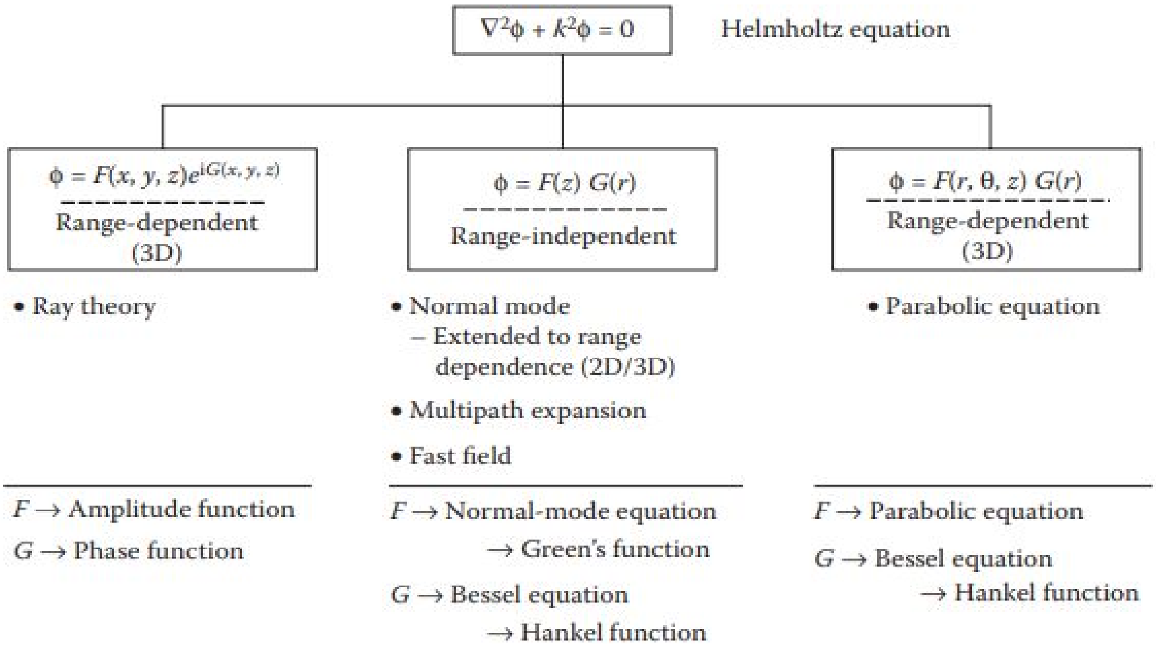


Figure 1. Summary of relationships among theoretical approaches for propagation modeling. (Adapted from Jensen, F.B. and Krol, H. 1975. The use of the parabolic equation method in sound propagation modelling. SACLANT ASW Res. Ctr, Memo. SM-72.)

In order to illustrate the relationships among the five approaches used to solve the wave equation, the rather elegant scheme developed by Jensen and Krol (1975) will be adopted with slight modifications [10].

Applications of Channel Modelling

Changes in the ocean soundscape have been driven by anthropogenic activity (e.g., naval-sonar systems, seismic-exploration activity, maritime shipping and windfarm development) and by natural factors (e.g., climate change and ocean acidification). New regulatory initiatives have placed additional restrictions on uses of sound in the ocean: mitigation of marine-mammal endangerment is now an integral consideration in acoustic-system design and operation. Applications may require high as well as low frequencies.

Low Frequency Applications: Applications of underwater sensing range from oil industry to aquaculture, and include instrument monitoring, pollution control, climate recording, prediction of natural disturbances, search and survey missions, and study of marine life [32]. Some are discussed here:-

International Monitoring System: Underwater acoustics plays a role in the international monitoring system (IMS), which comprises a network of stations that monitor Earth for evidence of nuclear explosions in all environments to ensure compliance with the Comprehensive Nuclear Test Ban Treaty (CTBT) [10]. The system employs seismic, hydro-acoustic and infrasound stations to monitor the underground, underwater, and atmosphere environments, respectively [32]. A lot can be known about the overall scenario of International waters.

Scientific Applications Observe the Environment: from geological processes on the ocean floor, to water characteristics (temperature, salinity, oxygen levels, bacterial and other pollutant content, dissolved matter, etc.) to counting or imaging animal life (micro-organisms, fish or mammals) [32].

Industrial Applications: to monitor and control commercial activities, such as underwater equipment related to oil or mineral extraction, underwater pipelines or commercial fisheries. Industrial applications often involve control and actuation components as well [33].

Military and Homeland Security: applications involve securing or monitoring port facilities or ships in foreign harbours, de-mining and communication with submarines and divers [33]. Seaweb [34] is an early example of a large deployable network for potential military applications. Its main goal was to investigate technology suitable for communication with and detection of submarines.

Water Quality Monitoring: It is important to monitor the quality of water. The underwater quality monitoring applications vary from monitoring water quality of canals to oceans. The authors in [35] have developed an application to monitor the quality of pool water for trout farms. For the growth of trout in a farm/pool, various parameters were monitored such as chemical oxygen demand, ammonium nitrogen (NH_3-N), pH, and electrical conductivity (EC).

Volcano, Earthquake, and Tsunami: These natural calamities can occur anytime and anywhere over the surface of earth and are even more alarming when they occur underwater, depending upon the seismic and the geological changes that take place under earth. Thus, it is important to monitor such conditions. Kumar et al. [35] have discussed UWSN architecture and proposed 4D-UWSN for early warning generation in case of any hazardous event such as earthquakes and tsunamis.

High Frequency Applications: Wireless information transmission through the ocean is one of the enabling technologies for the development of future ocean-observation systems and sensor networks [32].

Seismic Monitoring: A promising application for underwater sensor networks is seismic monitoring for oil extraction from underwater fields. Frequent seismic monitoring is of importance in oil extraction. Studies of variation in the reservoir over time are called “4-D seismic” and are useful for judging field performance and motivating intervention [36].

Equipment Monitoring and Control: Underwater equipment monitoring is a second example application. Long-term equipment monitoring may be done with pre-installed infrastructure. However, temporary monitoring would benefit from low-power, wireless communication [36].

Underwater Autonomous Robotic Applications: This includes coordinating adaptive sensing of chemical leaks or biological phenomena (for example, oil leaks or phytoplankton concentrations), and also equipment monitoring applications [36]. Underwater wireless sensing systems are envisioned for stand-alone applications and control of autonomous underwater vehicles (AUVs), and as an addition to cabled systems. For example, cabled ocean observatories are being built on submarine cables to deploy an extensive fibre-optic network of sensors (cameras, wave sensors and seismometers) covering miles of ocean floor [35].

Future Scope

Underwater channel has emerged as a promising technique due to low power consumption, compact size and better data rate. Despite of the benefits it offers, this technique has various lagging effects such as high bit error rate, battery power and bandwidth limitation and many more which need to be worked upon. Some means to combat those limitations and future work are discussed below:

Construction of Acoustic Propagation Model tailored for IOR (Indian Ocean Region): Construction of model specific to a particular region using one of the four approaches to underwater sound propagation namely rays, normal modes, Green’s function integral, and parabolic equation have been done for many areas such as: Baltic Sea [37], Arctic Sea [38, 39], Atlantic Ocean [40] and many others. India's location in the Indian Ocean Region (IOR) compels it to play a larger strategic role in the region [41]. Though a lot of work on underwater acoustics have been for IOR (several swell wave [42] propagation models [43] have also been worked upon), no models have been developed for acoustic propagation in Indian Ocean Region. Analysis of parameters of Indian Ocean can be done and then a validated low frequency propagation model can be developed.

Designing of Routing Protocols for Sensor Networks and make all existing ground-based routing protocols proactive: Underwater environment requires such protocols that are efficient in energy consumption, manage random variation in topology, and consider asymmetric links and huge propagation delay. DU et al. [44] present a protocol which is known as Level-Based Adaptive Geo-Routing (LBAGR) that divides communication traffic into four categories. These are upstream to sink, downstream to sensor nodes, downstream to specific nodes, and downstream to all nodes. T is protocol reduces communication end-to-end delays and improves delivery ratio and efficient utilization of battery power. Efficient utilization of battery power is the major concern of underwater sensor networks routing protocols [45].
Hybrid Technology: A combination of acoustic and optical channel model can be developed to improve the performance as both the technologies have their own advantages- Optical communication has higher bit rate, greater bandwidth and faster speed in water whereas acoustic communication suffers low loss in water and can cover large ranges [46]. Several attempts have been made to develop such technologies in different parts of the globe [47] and can also be developed for IOR.

Overcome the Performance Limitations Induced by the highly Dispersive Channel: In motion environments (such as platform motion of the moving sea surface and scattering), the slow propagation speed of sound introduces large Doppler spread or shifts, which causes severe interference among different frequency components of the signal (frequency spreading)[48]. On the outset, large Doppler spread results in a reduction in the channel coherence time or an apparent increase in the rate of channel fluctuation. Thus, the objective of underwater acoustic communication is to overcome the performance limitations induced by the highly dispersive channel.

- **References**

[1]Underwater acoustics. (2019, January 15). Retrieved May 24, 2019, from https://en.wikipedia.org/w/index.php?title=Underwater_acoustics&oldid=878559113
[2] speed of sound. (2019, April 7). Retrieved May 24, 2019, from https://en.wikipedia.org/w/index.php?title=Speed_of_sound&oldid=896088790t
[3] Newton, I. (1802). Mathematical principles of natural philosophy. A. Strahan.
[4] Bannon, Mike; Kaputa, Frank. "The Newton–Laplace Equation and Speed of Sound". Thermal Jackets. Retrieved 23 May 2019
[5] Finn, B. S. (1964). Laplace and the Speed of Sound. Isis, 55(1), 7-19.

[6] Anjum, Muhammad Ali Raza. "Underwater Acoustic Channel Modeling, Simulation, and Estimation." Proceedings of International Bhurban Conference on Applied Sciences & Technology Islamabad,Pakistan. 2011.

[7] Cañete, F. J., López-Fernández, J., García-Corrales, C., Sánchez, A., Robles, E., Rodrigo, F. J., & Paris, J. F. (2016). Measurement and Modeling of Narrowband Channels for Ultrasonic Underwater Communications. *Sensors (Basel, Switzerland)*, *16*(2), 256. doi:10.3390/s16020256

[8] TKACHENKO(2006),”Signal-to-noise ratio- an overview”,(science Direct)

[9] Santoso, Tri & Wirawan, Iwan & Hendrantoro, Gamantyo. (2013). Development of Underwater Acoustic Communication Model: Opportunities and Challenges. 2013 International Conference of Information and Communication Technology, ICoICT 2013. 10.1109/ICoICT.2013.6574601.

[10] P.C. Etter, Underwater Acoustic Modeling and Simulation, 3rd ed.,Spon Press, New York, 2003

[11] Ali, M. M., & Ramachandran, R. (Eds.). (2011). *Effect of Temperature and Salinity on Sound Speed in the Central Arabian Sea* (4th ed.). Retrieved from https://benthamopen.com/contents/pdf/TOOEJ/TOOEJ-4-71.pdf

[12] George, N. (2012, October 9). bathymetry. Retrieved June 3, 2019, from https://www.nationalgeographic.org/encyclopedia/bathymetry/

[13] Sindhu, B., Suresh, I., Unnikrishnan, A S , Bhatkar, N. V., Neetu, S., & Michael, G. S. (2007). Improved bathymetric datasets for the shallow water regions in the Indian Ocean. *Journal of Earth System Science*, *3*(116), 261–274.

[14] D. Christ, R., & L. WernliSr., R. (2014). Underwater Acoustics. *The ROV Manual*. Retrieved from https://www.sciencedirect.com/topics/earth-and-planetary-sciences/underwater-acoustics

[15] Sindhu, B., Suresh, I., Unnikrishnan, A S , Bhatkar, N. V., Neetu, S., & Michael, G. S. (2007). Improved bathymetric datasets for the shallow water regions in the Indian Ocean. *Journal of Earth System Science*, *3*(116), 261–274.

[16] Demopoulos, Amanda & Smith, Craig & A. Tyler, Paul. (2003). The deep Indian Ocean floor.

[17] A. Quazi and W. Konrad, “Underwater acoustic communications,” Communications Magazine, IEEE, vol. 20, no. 2, pp. 24 –30, march 1982.

[18] Stojanovic, M., & Beaujean, P.-P. (2011). Acoustic Communication [Epub]. In (4th ed.). Retrieved from

[19] Santoso, Tri & Wirawan, Iwan & Hendrantoro, Gamantyo. (2013). Development of Underwater Acoustic Communication Model: Opportunities and Challenges. 2013 International Conference of Information and Communication Technology, ICoICT 2013. 10.1109/ICoICT.2013.6574601.

[20] A. Lewin, P. (2010). Nonlinear acoustics in ultrasound metrology and other selected applications Author links open overlay panelPeter A.Lewin. *Physics Procedia*, *3*(1), 17–23. Retrieved from https://www.sciencedirect.com/science/article/pii/S1875389210000052

[21] The Physics Behind Acoustic Waves. (n.d.). Retrieved June 3, 2019, from https://www.universalclass.com/articles/science/physics/the-physics-behind-acoustic-waves.htm

[22] (2001). Uniform Plane Waves [Epub] (pp. 37–82). Retrieved from http://eceweb1.rutgers.edu/~orfanidi/ewa/ch02.pdf

[23] Sound Pressure - acoustic glossary - article. (n.d.). Retrieved June 3, 2019, from http://www.acoustic-glossary.co.uk/sound-pressure.htm

[24] A. Russel, D. (2012, May 17). Acoustics and Vibration Animations. Retrieved from https://www.acs.psu.edu/drussell/Demos/StandingWaves/StandingWaves.html

[25] Mills, D. (2016). Particle Velocity. *Pneumatic Conveying Design Guide*, *3*. Retrieved from https://www.sciencedirect.com/topics/engineering/particle-velocity

[26] Singh, K. (2017). Aluminium Oxide. *Applications of Targeted Nano Drugs and Delivery Systems*, *7*. Retrieved from https://www.sciencedirect.com/topics/engineering/aluminium-oxide

[27] Sakka, T., & H. Ogata, Y. (2005). Target Material. *Novel Materials Processing by Advanced Electromagnetic Energy Sources*. Retrieved from https://www.sciencedirect.com/topics/engineering/target-material

[28] L. Nedelec, S., & Campbell, J. (2016). Particle motion: the missing link in underwater acoustic ecology. *Methods in Ecology and Evolution*, *7*, 836–842. Retrieved from https://besjournals.onlinelibrary.wiley.com/doi/pdf/10.1111/2041-210X.12544

[29] 14.1 Speed of Sound, Frequency, and Wavelength | Texas Gateway. (n.d.). Retrieved June 3, 2019, from https://www.texasgateway.org/resource/141-speed-sound-frequency-and-wavelength

[30] The Laplace Operator. (n.d.). Retrieved June 3, 2019, from http://fourier.eng.hmc.edu/e161/lectures/gradient/node7.html

[31] Divergence. (n.d.). Retrieved June 3, 2019, from https://www.khanacademy.org/math/multivariable-calculus/multivariable-derivatives/divergence-and-curl-articles/a/divergence

[32] Comprehensive Nuclear-Test-Ban Treaty (CTBT) – UNODA. (n.d.). Retrieved June 3, 2019, from https://www.un.org/disarmament/wmd/nuclear/ctbt/

[33] Heidemann, J., Stojanovic, M., & Zorzi, M. (2011). Underwater sensor networks: applications, advances and challenges. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, *370*(1958), 158–175.

[34] Heidemann, J., Stojanovic, M., & Zorzi, M. (2012). Underwater sensor networks: applications, advances and challenges. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, *370*(1958), 158–175.

[35] Felemban, E., Shaikh, F. K., Qureshi, U. M., Sheikh, A. A., & Qaisar, S. B. (2015). Underwater Sensor Network Applications: A Comprehensive Survey. *International Journal of Distributed Sensor Networks*, *11*(11), 896832. https://doi.org/10.1155/2015/896832

[36] John Heidemann, J. H., Ye, W., Wills, J., Syed, A., & Li, Y. (2006). Research Challenges and Applications for Underwater Sensor Networking. Retrieved from https://www.isi.edu/~johnh/PAPERS/Heidemann06a.pdf

[37]Jakacki, J. & Meler, S. Ocean Dynamics (2019) 69: 1. https://doi.org/10.1007/s10236-018-1219-8

[38] Harrison, C. H. (1989). Ocean propagation models. *Applied Acoustics*, *27*(3), 163–201. https://doi.org/10.1016/0003-682X(89)90059-5

[39] Gavrilov, A. N. (2004). Low-frequency acoustic propagation loss in the Arctic Ocean: results of the Arctic Climate Observations using Underwater Sound experiment. *JASA Manuscript Submission System*.

[40] Buckingham, M. J. (1992). Ocean- Acoustic propagation Models. *Classification of Pysics*, 223–287.

[41] Das, A. (2014). New Perspective for Oceanographic Studies in the Indian Ocean Region. *Journal of Defense Studies*, *8*(1), 109–117. Retrieved from https://idsa.in/system/files/8_1_2014_NewPerspectiveforOceanographicStudiesintheIOR.pdf

[42] Access Denied. (n.d.). Retrieved June 1, 2019, from https://graphical.weather.gov/definitions/defineSwell.html

[43] Zheng, C. W., Li, C. Y., & Pan, J. (2018). Propagation Route and Speed of Swell in the Indian Ocean. *Journal of Geophysical Research: Oceans*, *123*(1), 8–21. https://doi.org/10.1002/2016JC012585

[44] X.-J. Du, K.-J. Huang, S.-L. Lan, Z.-X. Feng, and F. Liu, "LB-AGR: level-based adaptive geo-routing for underwater sensor network," Te Journal of China Universities of Posts and Telecommunications, vol. 21, no. 1, pp. 54–59, 2014.

[45] Awan, K. M., Shah, P. A., Iqbal, K., Gillani, S., Ahmad, W., & Nam, Y. (2019). Underwater Wireless Sensor Networks: A Review of Recent Issues and Challenges. *Wireless Communications and Mobile Computing*, *2019*, 1–20. https://doi.org/10.1155/2019/6470359

[46] Ankushe, S. S. (2015, April 19). Underwater channel modelling [Presentation slides]. Retrieved June 3, 2019, from https://www.slideshare.net/supriyaankushe5/underwater-channel-modelling-47163719

[47] Sampath , S. L., & Subashini, S. (2018). A HYBRID BASED MAC PROTOCOL FOR UNDERWATER ACOUSTIC SENSOR NETWORKS . *ARPN Journal of Engineering and Applied Sciences*, *13*(17), 4792–4799.

[48] Misra, A., & Gupta, D. (2013). CHALLENGES BEING FACED BY UNDERWATER WIRELESS SENSOR NETWORKS AND ITS COUNTERMEASURES. *International Journal Of Advance Research In Science And Engineering IJARSE*, *2*(9). Retrieved from https://pdfs.semanticscholar.org/7877/c024ca958c394c432bb68cb2efae39f4bfc5.pdf

[49] OELZE. (2005). The Acoustic Wave Equation and Simple Solutions. In (pp. 1–15). Retrieved from http://jontalle.web.engr.illinois.edu/uploads/473.F18/Lectures/Chapter_5a.pdf

[50] Bushman, V. A., & Fortov, V. E. (n.d.). EQUATION OF STATE. *A-to-Z Guide to Thermodynamics, Heat and Mass Transfer, and Fluids Engineering*. https://doi.org/10.1615/AtoZ.e.equation_of_state

[51] Specific Heat. (n.d.). Retrieved June 3, 2019, from http://hyperphysics.phy-astr.gsu.edu/hbase/thermo/spht.html

[52] Specific Heat | Boundless Physics. (n.d.). Retrieved June 3, 2019, from https://courses.lumenlearning.com/boundless-physics/chapter/specific-heat/

[53] Specific Heat at Constant Volume and Constant Pressure. (n.d.). Retrieved June 3, 2019, from https://www.nuclear-power.net/nuclear-engineering/thermodynamics/ideal-gas-law/specific-heat-constant-volume-pressure/

[54] The Laplace Operator. (n.d.-b). Retrieved June 3, 2019, from http://fourier.eng.hmc.edu/e161/lectures/gradient/node7.html

[55] Taylor Series -- from Wolfram MathWorld. (n.d.). Retrieved June 3, 2019, from http://mathworld.wolfram.com/TaylorSeries.html

[56] Padhi, Dr. R. (n.d.). NPTEL PHASE 2 - Courses [Presentation slides]. Retrieved June 3, 2019, from https://nptel.ac.in/courses/101108047/module7/Lecture+16.pdf

[57] Anderson, O. L. (1989). The relationship between the adiabatic bulk modulus and enthalpy for mantle-related minerals. *Physics and Chemistry of Minerals*, *16*(6), 559–662.
[58] (2000). *Adiabatic Bulk Moduli*. Retrieved from http://web.mit.edu/8.03-esg/watkins/8.03/bmod.pdf
[59] Mills, D. (2016b). Particle Velocity. *Pneumatic Conveying Design Guide*, *3*. Retrieved from https://www.sciencedirect.com/topics/engineering/particle-velocity
[60] Divergence. (n.d.-b). Retrieved June 3, 2019, from https://www.khanacademy.org/math/multivariable-calculus/multivariable-derivatives/divergence-and-curl-articles/a/divergence
[61] NPTEL :: Aerospace Engineering - Principles of Fluid Dynamics. (n.d.). Retrieved May 26, 2019, from https://nptel.ac.in/courses/101103004/12
[62] The curl of a gradient is zero - Math Insight. (n.d.). Retrieved June 3, 2019, from https://mathinsight.org/curl_gradient_zero
[63] World Web Math: Vector Calculus: Scalar Valued Functions. (n.d.). Retrieved June 2, 2019, from http://web.mit.edu/wwmath/vectorc/scalar/intro.html
[64] Frequency formula - sengpielaudio Sengpiel Berlin. (n.d.-b). Retrieved June 3, 2019, from http://www.sengpielaudio.com/calculator-period.htm

- **APPENDIX A**

❖ DERIVATION[49]:-

Mathematical expressions for some terms:-

1. **Particle position($\underline{r}$):** $\underline{r} = x\hat{x} + y\hat{y} + z\hat{z}$
2. **Particle displacement ($\underline{\xi}$):** $\underline{\xi} = \xi_x \hat{x} + \xi_y \hat{y} + \xi_z \hat{z}$
3. **Particle velocity($\underline{u}$):** $\underline{u} = \frac{\partial \xi}{\partial t}$
4. **Density:** ρ
5. **Condensation(S):** $s = \frac{\rho - \rho_0}{\rho_0}$

   ↓

   undisturbed (equilibrium) density
6. **Acoustic pressure:** $p = P - P_0 \rightarrow$ undisturbed (ambient pressure)

   ↓

   Instantaneous pressure

- **Three laws to develop wave equation for fluids are: -**

1) Equation of state:-

It is determined by thermodynamic properties relates changes in Pressure (P) and density (r). Dependent upon material (for example, an ideal gas is different from a liquid). We can expand the Equation of state into linear and nonlinear terms, but we will only be using the linear terms[50] .

An equation of state must relate three physical quantities describing the thermodynamic behavior of the fluid. Equation of state of an Ideal gas is : -

$$P = \rho r T_k$$

where p is the pressure in pascals, ρ is the density (Kg/m^3), r is the gas constant and T_K is the temperature in Kelvin. If the thermal conductivity is sufficiently low, the heat conduction during a cycle of the acoustic disturbance becomes negligible. In this case the condition is considered adiabatic and the relation between the pressure and density for the perfect gas is:

$$\frac{P}{P_0} = (\frac{\rho}{\rho_0})^{\gamma} \text{ where } \gamma(ratio\ of\ specific\ heats) = \frac{C_P}{C_V}$$

The **specific heat** is the amount of **heat** per unit mass required to raise the temperature by one degree Celsius. The relationship between **heat** and temperature change is usually expressed in the form shown below where c is the **specific heat**[51]. C_V represents the dimensionless **heat** capacity at **constant volume**; it is generally a function of temperature due to intermolecular forces[52]. C_P represents the dimensionless **heat** capacity at **constant pressure**; it is generally a function of temperature due to intermolecular forces[52]. Relation between them is explained in [53].

NOTE: $\square^2$ is Laplacian operator everywhere[54].

The taylor series expansion[55] of P in terms of ρ is as follows:-

$$P = P_0 + As + \frac{1}{2}Bs^2 + \frac{1}{3!}Cs^3 + ...$$

where $A = \rho_0(\frac{\partial P}{\partial \rho})_{\rho_0}$

$B = \rho_0{}^2(\frac{\partial^2 p}{\partial \rho^2})_{\rho_0}$

We linearize[56] the Taylor series by assuming small fluctuations so that only the lowest order term is retained to obtain:-

$$p = P - P_0 = As$$

where we define the coefficient A as the <u>adiabatic bulk modulus</u>, **B** = $\rho_0(\frac{\partial P}{\partial \rho})_{\rho_0}$

Bulk modulus is defined by. K = dP ÷ dv/v. It can be rewritten as K = dP÷dρ/ρ As the **Bulk modulus** is directly proportional to pressure, so **Bulk modulus** varies in an **adiabatic** process[57].Details can be accessed from [58].

$$\Rightarrow \frac{p}{\rho} \rightarrow \frac{m^2}{s^2}$$

$\Rightarrow \frac{p}{\rho} \rightarrow c^2$ (speed) (experimentally determined)

So, $p = \rho_0 c^2 s = Bs$ is the equation of state for linear acoustic waves in fluids (small changes in density $|s| \ll 1$).

2) The equation of continuity:-

Essentially it is conservation of mass. Relative Motion of fluid in a volume causes change in density[10].

Condensation , s, is fractional change in density and a generalized 3D equation of continuity is: $s = -\frac{\partial \xi_x}{\partial x} - \frac{\partial \xi_y}{\partial y} - \frac{\partial \xi_z}{\partial z}$

Or

$s = -(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}) \Rightarrow s = -\nabla \cdot \underline{\xi}$

If we differentiate S with respect to time we get:-

$s = -\nabla \cdot \underline{u}$ **(linear equation of continuity)**

Where $\underline{u} = \frac{\partial \xi}{\partial t}$

If we substitute for density we get,

$$\frac{\partial \rho}{\partial t} = -\rho_0 \nabla \cdot \underline{u}$$

3)The simple force equation(Euler’s Equation):-

Force Equation – Newton’s 2nd law

Pressure variations generate a force (F = P × Area) that causes particle motion[59]. The nonlinear inviscid force equation is:

$$-\nabla P = \rho[(\underline{u}\cdot\nabla)\underline{u} + \frac{\partial \underline{u}}{\partial t}]$$

To get the linear Euler's equation we will retain only the 1^{st} order terms:-

1^{st} : $p = P - P_0$ ⇒ $\Delta p = \Delta P$

2^{nd} : particle velocity is assumed to be small so second order terms are negligible. So we get:-

$$-\nabla p = \rho\frac{\partial \underline{u}}{\partial t}$$

3^{rd} : If we assume $|s \ll 1|$ (small) then $\rho = \rho_0$ finally gives us :-

$$-\nabla p = \rho_0\frac{\partial \underline{u}}{\partial t}$$ (linear Euler's equation, Newton's second Law)

- The linear wave equation:-

(1) Equation of State (Linearized):-

$$p = \rho_0 c^2 s = Bs$$

where **B** = $\rho_0(\frac{\partial P}{\partial \rho})_{\rho_0}$

(2) Continuity Equation (Linearized):-

$$\frac{\partial \rho}{\partial t} = -\rho_0 \nabla\cdot\underline{u}$$

(3) Euler's Equation (Linearized):-

$$-\nabla p = \rho_0\frac{\partial \underline{u}}{\partial t}$$

To derive the Linear Wave Equation we first take the derivative of s

Recall : $s = \frac{\rho - \rho_0}{\rho_0}$

So that $\frac{ds}{dt} = \frac{1}{\rho_0}\frac{d\rho}{dt}$

or $\frac{d\rho}{dt} = \rho_0\frac{ds}{dt}$

Substituting this into the Continuity Equation (2) gives the relative change in density as the divergence of the particle velocity:-

$$\rho_0\frac{\partial s}{\partial t} = -\rho_0\nabla\cdot\vec{u}$$

Substituting in equation of state (1) yields:-

$$\frac{\partial}{\partial t}[\frac{p}{B}] = -\nabla\cdot\underline{u}$$

$$P\frac{\partial}{\partial T} = -B\nabla\cdot\underline{u} \qquad (4)$$

Taking the derivative of (4) wrt time gives:-

$$\frac{\partial^2 p}{\partial t^2} = -B\frac{\partial}{\partial t}\nabla\cdot\underline{u} = -B\nabla\cdot\frac{\partial \underline{u}}{\partial t} \qquad (5)$$

If we take divergence[60] of Euler's equation(3), we get:-

$$\nabla\cdot[-\nabla p = \rho_0\frac{\partial \underline{u}}{\partial t}]$$

$$-\frac{1}{\rho_0}\nabla^2 p = \nabla\cdot\frac{\partial \underline{u}}{\partial t}$$

Substituting into (5) gives the result:-

$$\frac{\partial^2 p}{\partial t^2} = -B\times-\frac{1}{\rho_0}\nabla^2 p$$

$$\frac{\partial^2 p}{\partial t^2} = \frac{B}{\rho_0}\nabla^2 p$$

which is the classical **wave equation**.

At adiabatic conditions, bulk modulus **B =** $\rho_0 c^2$ **(Newton-Laplace equation[6]), c is sound speed.**

Hence by using this we get Linearized Wave Equation:-

$$\frac{\partial^2 p}{\partial t^2} = c^2 \nabla^2 p \qquad \textbf{(6)}$$

We can also define a velocity potential (similar to EM)[61].

From eq (3), Euler's equation, we note that the curl of a gradient is zero($\nabla\times\nabla \underline{f} = 0$)[62] so:

$$\nabla\times\{-\nabla p = \rho_0 \frac{\partial \underline{u}}{\partial t}\}$$

$$-\nabla\times\nabla p = 0 = \rho_0 \frac{\partial \nabla\times\underline{u}}{\partial t}$$

which implies that

$$\nabla\times\underline{u} = 0$$

This means that the particle velocity can be expressed as the gradient of a scalar function[63]:-

$$\underline{u} = \nabla\Phi$$

where Φ is a scalar function.

Substituting back into the Euler's equation gives: $-\nabla p = \rho_0 \frac{\partial\nabla\Phi}{\partial t}$

or the relation between the pressure and the velocity potential is:

$$p = -\rho_0 \frac{\partial \Phi}{\partial t}$$

Appendix B[49]

❖ Helmholtz equation from wave equation

The velocity potential, Φ **, can also be shown to satisfy the wave equation(6):-**

$$\nabla^2\Phi = \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}$$

Assuming the harmonic solution $\Phi = \varphi e^{-i\omega t}$ where , φ is the time-independent potential function, in the above equation we get[10]:-

$$\nabla^2\Phi = \frac{1}{(\lambda\times f)^2}\frac{\partial^2(\varphi e^{-i\omega t})}{\partial t^2}$$

As sound speed $c = \lambda\times f$[64].

$$\nabla^2\Phi = \frac{1}{(\lambda\times f)^2}\frac{\partial(-\varphi i\omega e^{-i\omega t})}{\partial t}$$

$$\Rightarrow \qquad \nabla^2\Phi = \frac{1}{(\lambda\times f)^2}(-\varphi(i\omega)^2 e^{-i\omega t})$$

$$\Rightarrow \qquad \nabla^2\Phi = \frac{-1}{(\lambda\times f)^2}(\varphi\omega^2 e^{-i\omega t})$$

Angular frequency , $\omega = 2\Pi f$[64] and Wave number, $k = \frac{2\pi}{\lambda}$ [64]

$$\Rightarrow \qquad \nabla^2\Phi = \frac{-1}{(\lambda\times f)^2}(\varphi(2\pi f)^2 e^{-i\omega t})$$

$$\Rightarrow \qquad \nabla^2\Phi = \frac{-(2\Pi)^2}{(\lambda)^2}(\varphi e^{-i\omega t})$$

$$\Rightarrow \qquad \nabla^2\Phi = -k^2(\varphi e^{-i\omega t})$$

$$\Rightarrow \qquad \nabla^2\Phi + k^2(\varphi e^{-i\omega t}) = 0$$

$$\Rightarrow \qquad \nabla^2(\varphi e^{-i\omega t}) + k^2(\varphi e^{-i\omega t}) = 0$$

$$\Rightarrow \qquad \nabla^2\varphi + k^2\varphi = 0$$

↖

Helmholtz Equation

Note that this equation has no explicit time dependence.